But, $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$$\therefore \quad m(t) = \frac{1}{4} V_m V_c \{\cos[2\pi(2f_c + f_m)t] + \cos(2\pi f_m t)\} + \frac{1}{4} V_m V_c$$
$$\{\cos[2\pi(2f_c - f_m)t] + \cos(2\pi f_m t)\} \quad \text{... (3.106)}$$

The terms having frequencies $(2f_c + f_m)$ and $(2f_c - f_m)$ are removed by the low pass filter which follows the product modulator. But, the terms having frequency $f_m$ are passed through to the output.

**2. Output Voltage**

$$\text{Output voltage } v_0(t) = \frac{1}{2} V_m V_c \cos(2\pi f_m)t \quad \text{... (3.107)}$$

Thus, the scaled message signal is obtained at the detector output.

## 3.28 SINGLE SIDEBAND SUPPRESSED-CARRIER (SSB-SC) MODULATION *(Important)*

**1. Definition**

As discussed earlier that amplitude modultion and double-sideband suppressed-carrier (DSB-SC) modulation are wasteful of bandwidth since they both need a transmission bandwidth equal to twice the message signal bandwidth. In either case one half of the transmission bandwidth is occupied by the upper sideband of the modulated signal whereas the other half is occupied by the lower sideband. However, the lower and upper sidebands are uniquely related to each other by virtue of their symmetry about the carrier frequency, *i.e.*, if amplitude and phase spectra of either sideband is given, we can uniquely determine the other. This means that as far as the transmission of information is concerned, only one sideband is necessary. Thus, if the carrier and one of the two sidebands are suppressed at the transmitter, no information is lost. Modulation of this type which provides a single sideband with suppressed carrier is known as single sideband suppressed carrier (SSB-SC) system. Thus, SSB-SC system reduces the transmission bandwidth by half. This means that in a given frequency band we can accommodate twice the number of channels by using a single sideband in place of both the sidebands*.

**2. Frequency Spectrum**

Figure 3.38 further illustrate the concept of single sideband modulation with the help of different frequency spectrums.

> **REMEMBER**
>
> For SSB transmissions, it does not matter whether the upper or lower sideband is used, since the information is contained in both.

Figure 3.38($a$) shows the frequency spectrum of modulating or baseband signal. It contains $\omega_m$ as the maximum frequency component. Figure 3.38($b$) shows the frequency spectrum of a DSB-SC modulation which contains no carrier but two sidebands, *i.e.*, lower sideband and upper sideband. Figure 3.38($c$) shows the frequency spectrum of single sideband suppressed carrier modulation consisting of upper sideband only, *i.e.*, lower sideband and carrier signal are suppressed. Figure 3.38($d$) shows the frequency spectrum of single sideband suppressed carrier modulation consisting of lower sideband only, *i.e.*, upper sideband and carrier signal are suppressed.

* An AM signal with no carrier and one sideband is called a single sideband (SSB) signal. The upper and lower sidebands contain the same information, and one is not preferred over the other.

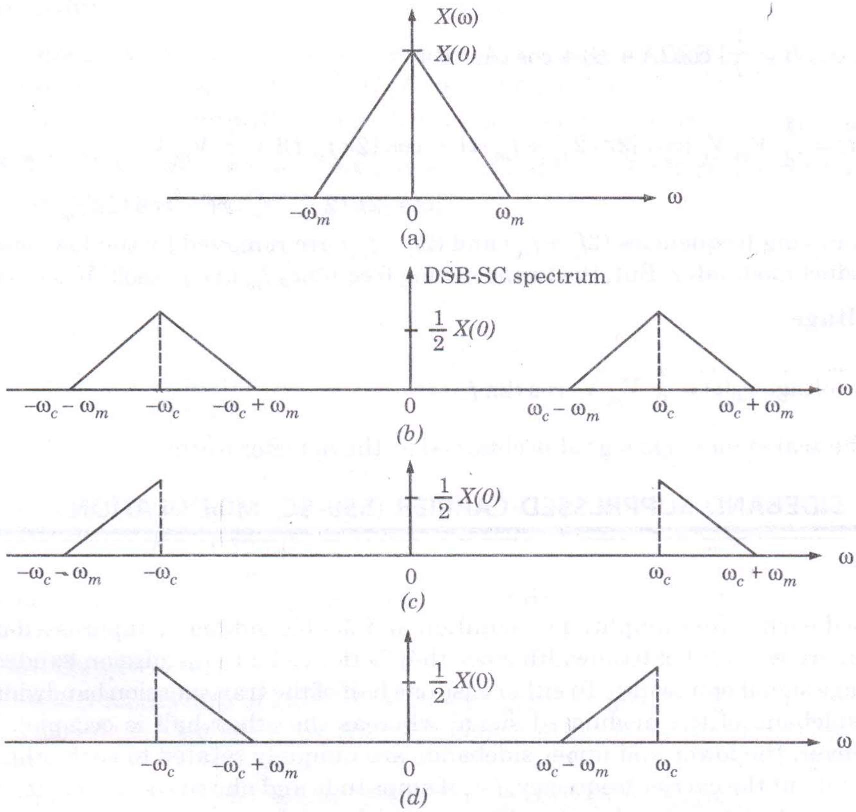


**Fig. 3.38** *(a) Spectrum of baseband signal (b) Spectrum of DSB-SC wave (c) Spectrum of SSB-SC wave with the upper sideband transmitted (d) Spectrum of SSB-SC wave with the lower sideband transmitted.*

### 3.28.1. Time-Domain Description of the SSB-SC Wave

We can derive an expression to represent the SSB wave in time-domain considering a special case of single-tone modulating signal. We can derive a general expression for SSB wave using the concept of a pre-envelope

**1. SSB-SC wave with single-tone modulating signal**

Let us consider a single-tone modulating signal as

$$x(t) = \cos \omega_m t \qquad \text{...(3.108)}$$

The frequency spectrum of this modulating signal consist of two impulses located at $\omega = \pm\, \omega_m$ as shown in figure 3.39(*a*). If this modulating signal, modulates a carrier signal $\cos \omega_c t$, then the resulting spectrum of DSB-SC signal will be as shown in figure 3.39 (*b*).

To get the SSB-SC waveform, we will have to eliminate one of the two sidebands.

Figure 3.39(*c*) shows the spectrum of SSB-SC wave with lower sidebands. From this figure, it is clear that this spectrum corresponds to a time-domain signal $\cos(\omega_c - \omega_m)t$, because the frequency spectrum of cosine function contains two impulses in its frequency domain. This means that a SSB-SC wave with lower sideband may be expressed as

$$\cos(\omega_c - \omega_m)t = \cos \omega_m t \cos \omega_c t + \sin \omega_m t \sin \omega_c t \qquad \text{...(3.109)}$$

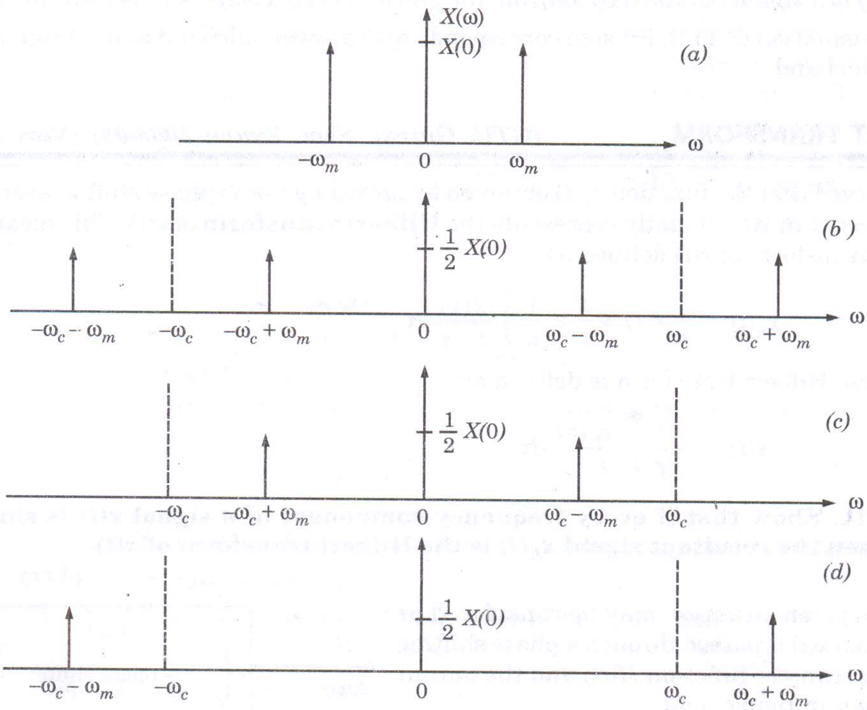


**Fig. 3.39** *(a) Frequency spectrum of modulating signal (b) Frequency spectrum of DSB-SC wave (c) Spectrum of SSB-SC with lower sideband, (d) Spectrum of SSB-SC with upper sideband.*

In the same manner, the expression for the single-tone SSB-SC wave with upper sideband may be expressed as

$$\cos(\omega_c + \omega_m)t = \cos \omega_m t \cos \omega_c t - \sin \omega_m t \sin \omega_c t \qquad ...(3.110)$$

Both these equations (3.109) and (3.110) may be combined as under :

$$s(t)_{SSB} = \cos \omega_m t \cos \omega_c t \pm \sin \omega_m t \sin \omega_c t \qquad ...(3.111)$$

Here, the (+) sign represents the lower sideband and (–) sign represented the upper sideband.

We may write the terms $\sin \omega_c t$ and $\sin \omega_m t$ as under:

$$\sin \omega_c t = \cos\left(\omega_c t - \frac{\pi}{2}\right) \qquad ...(3.112(a))$$

$$\sin \omega_m t = \cos\left(\omega_m t - \frac{\pi}{2}\right) \qquad ...(3.112\,(b))$$

This means that the sine terms may be obtained using the corresponding cosine terms simply by giving a phase shift of (–π/2). The expression in equation (3.112 (*b*)) represents the SSB-SC wave for the case of single-tone modulation, however, it provides a way to get a general expression for SSB-SC wave. In the expression of equation (3.111), the term $\sin \omega_m t$ is obtained by giving a phase shift of (–π/2) to the modulating frequency $\cos \omega_m t$. Similarly, in a general modulating signal $x(t)$, if all the frequency components are shifted by (–π/2), it may lead to a general expression of SSB-SC signal. As this point, it may be noted that $x(t)$ may be expressed as a continuous sum of sinusoidal signals. Hence, expression in equation (3.111) may be extended for a SSB-SC signal modulated by a general modulating signal $x(t)$ as expressed below :

$$s(t)_{SSB} = x(t) \cos \omega_c t \pm x_h(t) \sin \omega_c t \quad ...(3.113)$$

*where $x_h(t)$ is a signal obtained by shifting the phase of every component present in $x(t)$ by $(-\pi/2)$*

Similar to equation (3.111), (+) sign corresponds to the lower sideband and (–) sign corresponds to the upper sideband.

## 3.29 HILBERT TRANSFORM *(GTU, Gujrat, Sem. Exam; 2009-08) (Very Important)*

It may be observed that the function $x_h(t)$ obtained by providing $(-\pi/2)$ phase shift to every frequency component present in $x(t)$, actually represents the **Hilbert transform** of $x(t)$. This means that $x_h(t)$ is the Hilbert transform of $x(t)$ defined as

$$x_h(t) = \frac{1}{\pi} x(t) \otimes \frac{1}{t} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau \quad ...(3.114)$$

Also, the inverse Hilbert transform is defined as

$$x(t) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x_h(\tau)}{t-\tau} d\tau \quad ...(3.115)$$

**EXAMPLE 3.14. Show that if every frequency component of a signal $x(t)$ is shifted by an amount $\pi/2$ then the resultant signal $x_h(t)$ is the Hilbert transform of $x(t)$.**

*(Very Important)*

**Solution:** The given situation may be considered as though the signal $x(t)$ is passed through a phase shifting system having transfer function $H(\omega)$ and the output is $x_h(t)$ as shown in figure 3.40.

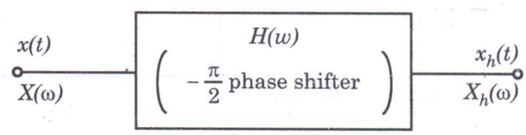


**Fig. 3.40** *A phase shifting system*

The characteristics of this system may be specified as under :

(*i*) The magnitude of the frequency components present in $x(t)$ remains unchanged when it is passed through the system. This means that $H(\omega) = 1$, and

(*ii*) The phase of the positive frequency components is shifted by $-\frac{\pi}{2}$. Now, since the phase spectrum $\theta(\omega)$ has an odd symmetry, the phase of the negative frequency components is shifted by $+\frac{\pi}{2}$. The spectrums $H(\omega)$ and $\theta(\omega)$ have been plotted in figure 3.41.

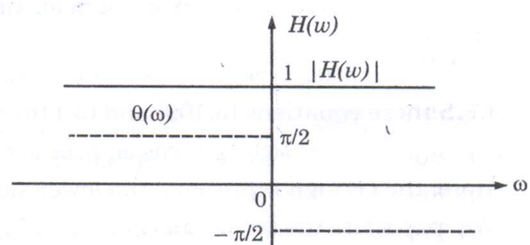


**Fig. 3.41** *Transfer function of $-\pi/2$ phase shifter*

The transfer function is expressed as

$$H(\omega) = |H(\omega)| e^{j\theta(\omega)}$$

or

$$H(\omega) = 1.e^{j\theta}(\omega) \quad ...(3.116)$$

From figure 3.41, it may be observed that

$$\theta(\omega) = \begin{cases} +\frac{\pi}{2} \text{ for } \omega < 0 \ \ (i.e., \text{negative frequencies}) \\ -\frac{\pi}{2} \text{ for } \omega > 0 \ \ (i.e., \text{ positive frequencies}) \end{cases} \quad ...(3.117)$$

Therefore equation (3.116) may be modified as

$$H(\omega) = \begin{cases} e^{j\pi/2} \text{ for } \omega < 0 \\ e^{-j\pi/2} \text{ for } \omega > 0 \end{cases} \quad ...(3.118)$$

Also, we know that

$$e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2}$$

or $$e^{j\frac{\pi}{2}} = j$$

and $$e^{-j\frac{\pi}{2}} = \cos\left(-\frac{\pi}{2}\right) + j\sin\left(-\frac{\pi}{2}\right)$$

or $$e^{-j\frac{\pi}{2}} = -j$$

**DO YOU KNOW?**

Pure SSB signals are used in telephone systems as well as in two-way radio. Two-way SSB communications is used in the military, in CB radio, and by hobbyists known as radio amateurs.

Thus, $H(\omega)$ becomes

$$\frac{H(\omega)}{j} = \begin{cases} 1 & \text{for } \omega < 0 \\ -1 & \text{for } \omega > 0 \end{cases} = -\,sgn\,(\omega)$$

or $$H(\omega) = -j\,sgn\,(\omega) \qquad ...(3.119)$$

The response $X_h(\omega)$ of the phase shifting system is related to the input $X(\omega)$ as

$$X_h(\omega) = X(\omega) \cdot H(\omega) \qquad ...(3.120)$$

where $$x(t) \longleftrightarrow X(\omega)$$

and $$x_h(t) \longleftrightarrow X_h(\omega)$$

Now, substituting the value of $H(\omega)$ in equation (3.119) from equation (3.120) we get

$$X_h(\omega) = -jX(\omega)\,sgn\,(\omega)$$

Taking the inverse Fourier transform of both sides of last equation, we get

$$x_h(t) = F^{-1}[-j.X(\omega)\,sgn(\omega)]$$

Also the time-domain of $sgn(\omega)$ is given as

$$\frac{1}{\pi t} \leftrightarrow sgn(\omega)$$

Using time convolution theorem, we get

$$x_h(t) = \frac{1}{\pi}\left[x(t) \otimes \frac{1}{t}\right] = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}\,d\tau$$

which is the Hilbert transform of $x(t)$.

### 3.29.1. Some applications of the Hilbert transform may be listed as under:

(*i*) For generation of SSB signals,

(*ii*) For designing of minimum phase type filters,

(*iii*) For representation of bandpass signals.

### 3.29.2. Properties of the Hilbert Transform

Following are the properties of Hilbert transform:

(*i*) A signal $x(t)$ and its Hilbert transform $x_h(t)$ have the same energy density spectrum.

(*ii*) A signal $x(t)$ and its Hilbert transform $x_h(t)$ have the same autocorrelation function.

(*iii*) A signal $x(t)$ and its Hilbert transform $x_h(t)$ are mutually orthogonal. Mathematically,

$$\int_{-\infty}^{\infty} x(t)x_h(t)\,dt = 0 \qquad ...(3.121)$$

(*iv*) If $x_h(t)$ is a Hilbert transform of $x(t)$, then the Hilbert transform of $x_h(t)$ is $-x(t)$, *i.e.*,

If $$H[x(t)] = x_h(t)$$

then $H[x_h(t)] = -x(t)$

Here '$H$' denotes the Hilbert transform.

### 3.29.3. The Pre-envelope or Analytic Signal

The concept of pre-envelope, also called as the analytic function is quite useful in deriving the general expression of the SSB-SC signal.

The pre-envelope of a real-valued signal $x(t)$ is defined as

$$x_p(t) = x(t) + jx_h(t) \quad ...(3.122)$$

where $x_h(t)$ is the Hilbert transform of signal $x(t)$. Clearly the pre-envelope $x_p(t)$ is a complex-valued signal. The real part of $x_p(t)$ is $x(t)$, and the imaginary part is its Hilbert transform $x_h(t)$. The complex conjugate of the pre-envelope denoted by $x_p(t)$ is expressed as

$$x_p^*(t) = x(t) - jx_h(t) \quad ...(3.123)$$

## 3.30 SSB-SC FOR A GENERAL MODULATING SIGNAL

### 1. Definition and Mathematical Expressions

In previous section we have already derived an expression for a generalized SSB-SC signal by extending the concept of SSB-SC for a signal-tone modulation. In this article let us derive the same expression (*i.e.*, equation) by using the concept of pre-envelope or analytic signal.

As we know the pre-envelope of a function $x(t)$ is defined as

$$x_p(t) = x(t) + jx_h(t) \quad ...(3.124)$$

The Fourier transform of $x_p(t)$ is the sum of the Fourier transforms of $x(t)$ and $x_h(t)$ *i.e.*,

$$F[x_p(t)] = F[x(t)] + jF[x_h(t)]$$

or $$X_p(\omega) = X(\omega) + j[-j X(\omega)\, sgn(\omega)]$$

or $$X_p(\omega) = X(\omega) + X(\omega)\, sgn(\omega) \quad ...(3.124a)$$

We know that $$sgn(\omega) = \begin{cases} 1 & \text{for } \omega > 0 \\ -1 & \text{for } \omega < 0 \end{cases}$$

Therefore, we have

$$X_p(\omega) = \begin{cases} 2X(\omega) & \text{for } \omega > 0 \\ 0 & \text{for } \omega < 0 \end{cases} \quad ...(3.125)$$

Figures 3.42($a$) and ($b$) show $X(\omega)$ and $X_p(\omega)$ respectively. It is obvious from figure 3.42($b$) that $X_p(\omega)$ vanishes for negative frequencies.

Similarly, we can find the Fourier transform of $x_p^*(t)$ defined by equation (3.123) as under:

$$X_p^*(\omega) = X(\omega) - j[-j \cdot X(\omega)\, sgn(\omega)]$$

or $$X_p^*(\omega) = X(\omega) - X(\omega)\, sgn(\omega)$$

which may be written as

$$X_p^*(\omega) = \begin{cases} 0 & \text{for } \omega > 0 \\ 2X(\omega) & \text{for } \omega < 0 \end{cases}$$

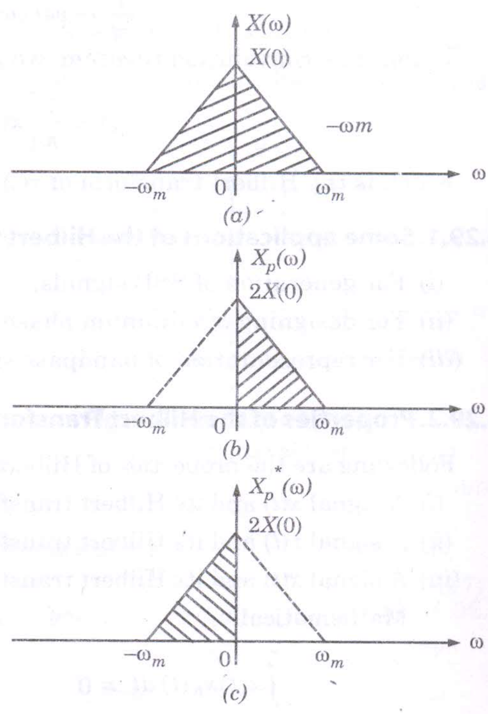

Fig. 3.42.

This has been plotted in figure 3.42($c$). Clearly, $X_p^*(\omega)$ vanishes for positive frequencies. Now, let us take an SSB-SC wave consisting of only the lower sidebands of a general modulating signal $x(t)$ as shown in figure 3.42 (*d*). Following points may be observed from figure 3.42 (*d*).

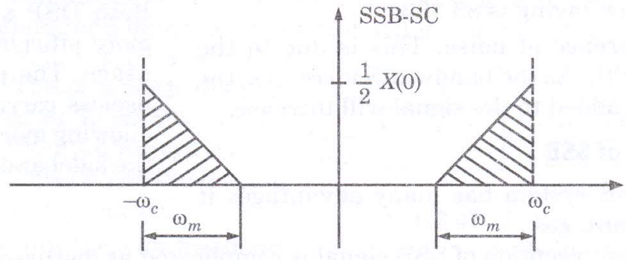


**Fig. 3.42** *(d) A SSB-SC spectrum consisting of lower sideband only*

(*i*) The right-hand portion of the figure represents a spectrum of $\frac{1}{4}x_p^*(t)\, e^{j\omega_c t}$. This is equivalent to shifting the spectrum $X_p^*(\omega)$ towards right by $\omega_c$.

(*ii*) Similarly, the left-hand portion of figure 3.42(*d*) represents the spectrum of $\frac{1}{4}x_p(t)\, e^{-j\omega_c t}$.

Thus, figure 3.42(*d*) represents the spectrum of a combined signal $x_p(t)\, e^{+j\omega_c t} + x_p(t)\, e^{-j\omega_c t}$. Therefore, the time-domain representation of the SSB-SC spectrum shown in figure 3.42 is expressed as

> **REMEMBER**
> Among the family of AM systems, SSB modulation is optimum with regard to noise performance as well as band-width conservation.

$$s(t)_{SSB} = \frac{1}{4}[x_p^*(t)\, e^{j\omega_c t} + x_p(t)\, e^{-j\omega_c t}]$$

Substituting in terms of Hilbert transform from equations (3.123) and (3.124), we get

$$s(t)_{SSB} = \frac{1}{4}[x(t) - jx_p(t)]\, e^{j\omega_c t} + \frac{1}{2}[x(t) + jx_h(t)]\, e^{-j\omega_c t}$$

or $$s(t)_{SSB} = \frac{1}{2}x(t)\left[\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}\right] + \frac{j}{2}x_h(t)\left[\frac{e^{-j\omega_c t} - e^{j\omega_c t}}{2}\right]$$

or $$s(t)_{SSB} = \frac{1}{2}[x(t)\cos\omega_c t + x_h(t)\sin\omega_c t] \qquad ...(3.126)$$

which is the time-domain description of the SSB-SC signal consisting of only the lower sidebands.

Similarly we can derive an expression for an SSB-SC signal consisting of the upper sidebands as under :

$$s(t)_{SSB} = \frac{1}{2}[x(t)\cos\omega_c t - x_h(t)\sin\omega_c t] \qquad ...(3.127)$$

Hence, the time-domain description of the SSB-SC wave is represented by the expression as under :

$$s(t)_{SSB} = x(t)\cos\omega_c t \pm x_h(t)\sin\omega_c t \qquad ...(3.128)$$

where '+' and '–' signs corresponds to the lower sidebands and upper sidebands respectively.

### 3.30.1. Advantages of SSB over DSB-FC*

The advantages of SSB over DSB-FC signal are :

(*i*) Less bandwidth requirements as SSB requires a BW of $f_m$. This will allow more number of signals to be transmitted in the same frequency range.

* DSB AM is not widely used. However, SSB is widely used in two-way radio communications.

(ii) Lots of power saving. This is due to the transmission of only one sideband component. At 100% modulation, the percent power saving is 83.33%.

(iii) Reduced interference of noise. This is due to the reduced bandwidth. As the bandwidth increases, the amount of noise added to the signal will increase.

**DO YOU KNOW?**

Both DSP and SSB signals are more efficient in terms of power usage. The power wasted in the useless carrier is saved, thereby allowing more power to be put into the sidebands.

### 3.30.2. Disadvantages of SSB

Even though the SSB system has many advantages it has the following disadvantages :

(i) The generation and reception of SSB signal is complicated as discussed later on.

(ii) The SSB transmitter and receiver need to have an excellent frequency stability. A slight change in frequency will hamper the quality of transmitted and received signal. Therefore, SSB is not generally used for the transmission of good quality music. It is used for speech transmission.

### 3.30.3. Applications of SSB

(i) SSB transmission is used in the applications where the power saving and low bandwidth requirements are important.

(ii) The application areas are land and air mobile communication, telemetry, military communications, navigation and amateur radio. Many of these applications are point to point communication applications.

**EXAMPLE 3.15. Calculate the percent power saving for the SSB signal if the AM wave is modulated to a depth of (a) 100% and (b) 50%.**

**Solution : Power Saving in SSB Signal :** Carrier and one sideband are suppressed. Therefore, only one sideband is transmitted.

$$\text{Therefore,} \quad \%\text{ power saving} = \frac{\text{Power in carrier + Power in one sideband}}{\text{Total Power}}$$

$$\text{or} \quad \%\text{ power saving} = \frac{P_c\left[1+\frac{m^2}{4}\right]}{P_c\left[1+\frac{m^2}{2}\right]} = \frac{\left[1+\left(\frac{m^2}{4}\right)\right]}{\left[1+\left(\frac{m^2}{2}\right)\right]} \quad \text{... (i)}$$

At 100% modulation, $m = 1$

$$\%\text{ power saving} = \frac{1.25}{1.5} = 83.33\% \quad \text{... (ii)}$$

At 50% modulation, $m = 0.5$

$$\text{Therefore,} \quad \%\text{ power saving} = \frac{1.0625}{1.125} = 94.44\% \quad \text{... (iii)}$$

## 3.31 GENERATION OF SSB-SC SIGNAL*

SSB-SC signals may be generated by two methods as under:

(i) Frequency discrimination method or filter method

(ii) Phase discrimination method or phase-shift method

* The most common way of generating an SSB signal is to use the filter method which incorporates balanced modulator followed by a highly selective filter that passes either the upper or lower sideband.

### 3.31.1. Frequency Discrimination Method

In a frequency discrimination method, firstly, a DSB-SC signal is generated simply by using an ordinary product modulator or a balanced modulator. After this, from the DSB-SC, signal one of the two sidebands is filtered out by a suitable bandpass filter (BPF). The schematic diagram for this method is shown in figure 3.43. Infact, the design of bandpass filter is quite critical and thus puts same limitations on the modulating or baseband and carrier frequencies.

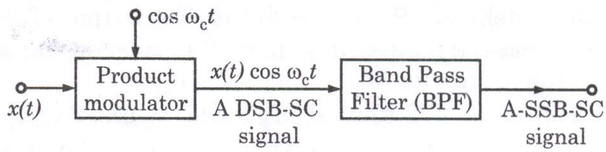


**Fig. 3.43.** *Frequency-discrimination method for SSB-SC generation.*

Following are the limitations for frequency discrimination method:

(*i*) The frequency-discrimination method is useful only if the baseband signal is restricted at its lower edge due to which the upper and lower sidebands are non-overlapping. For example, the filter method is used for speech communication where lowest spectral component is 70 Hz and it may be taken as 300 Hz without affecting the intelligibility of the speech signal. However, the system is not useful for video communication where the baseband signal starts from dc.

(*ii*) The another restriction of the frequency discrimination method is that the baseband signal must be appropriately related to the carrier frequency. Infact, the design of the bandpass filter (BPE) becomes difficult if the carrier frequency is quite higher than the bandwidth of the baseband signal.

### 3.31.2. Phase Shift Method for the SSB Generation*

***(JNTU, Hyderabad, Sem. Exam; 2003-04, 2004-2005, 2006-07)***

**1. Definition and Block Diagram**

Figure 3.44 shows the block diagram for the phase shift method of SSB generation. This system is used for the suppression of lower sideband. This system uses two balanced modulators $M_1$ and $M_2$ and two 90° phase shifting networks as shown in figure 3.44.

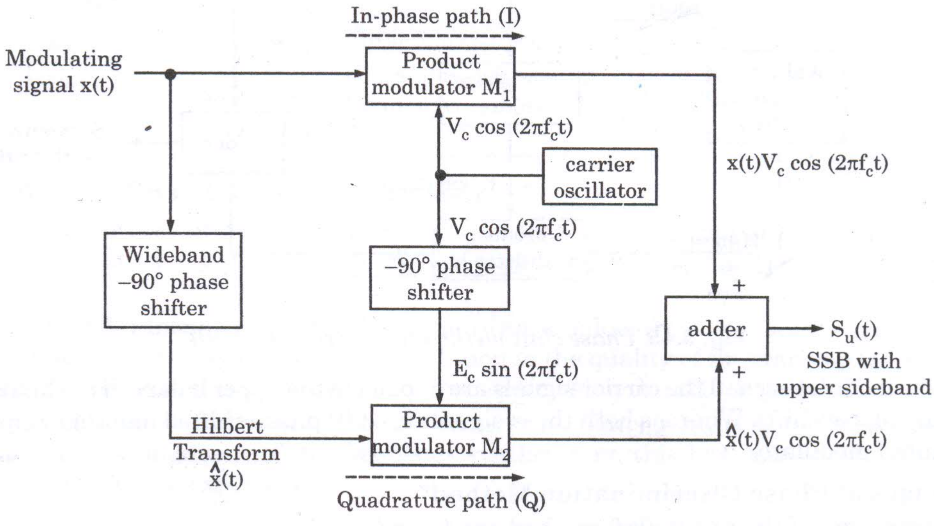


**Fig. 3.44** *Phase discrimination method for generating SSB signals*

* The phasing method of SSB generation uses two balanced modulators and 90° phase shifters for the carrier and modulating signal to produce two DSB signals that, when added, cause one sideband to be canceled out.

**2. Working Operation**

The message signal $x(t)$ is applied directly to the product modulator $M_1$ and through a – 90° phase shifter to the product modulator $M_2$. Hence, we get the Hilbert transform $\hat{x}(t)$ at the output of the wideband – 90° phase shifter. The output of carrier oscillator is applied as it is to modulator $M_1$ whereas it is passed through a – 90° phase shifter and applied to the modulator $M_2$.

**DO YOU KNOW?**

Most SSB filters are made with quartz crystals.

Output of $M_1 = x(t) \cdot V_c \cos(2\pi f_c t)$

and output of $M_2 = \hat{x}(t) \cdot V_c \sin(2\pi f_c t)$

The outputs of $M_1$ and $M_2$ are applied to an adder. Note the (–) sign for the quadrature path

Therefore,

$$\text{Adder output} = x(t) \cdot V_c \cos(2\pi f_c t) + \hat{x}(t) \cdot E_c \sin(2\pi f_c t)$$

or
$$\text{Adder output} = V_c\,[x(t) \cdot \cos(2\pi f_c t) + \hat{x}(t) \cdot V_c \sin(2\pi f_c t)] \quad \text{... (3.132)}$$

This expression represents the SSB signal with only USB i.e. it rejects the LSB.

## 3.32 SSB WITH LOWER SIDEBAND (LSB)

The block diagram of the phase discriminator method to generate SSB with LSB is shown in figure 3.45. Note that the adder polarities for the in-phase and quadrature paths are both positive.

**Suppression of the upper sideband**

We can suppress the USB and generate the SSB signal consisting of the LSB by arranging the blocks as shown in figure 3.45.

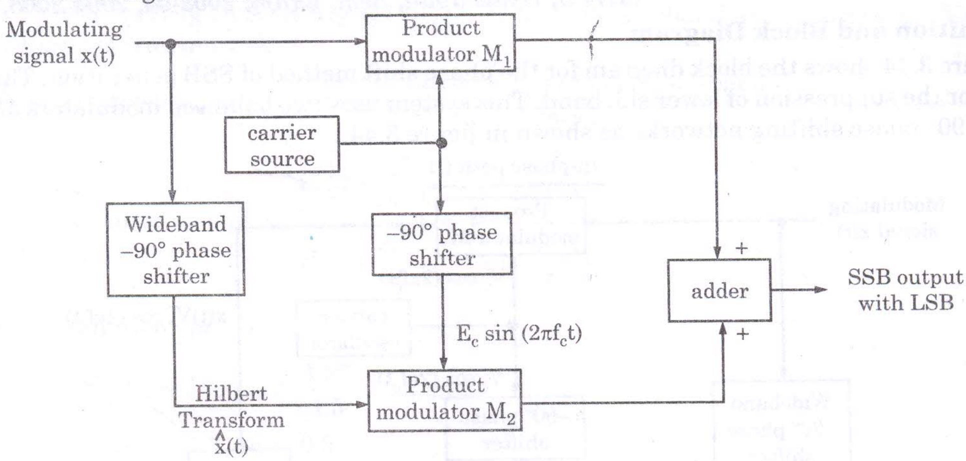


**Fig. 3.45** *Phase shift method to suppress the USB*

Here, the modulating and the carrier signals are applied to the upper balanced modulator directly (without any phase shift). Whereas both these signals are 90° phase shifted and then applied to the lower balanced modulator.

**2. Advantages of Phase Discrimination Method***

The advantages of the phase shift method are as under:

(*i*) It can generate the SSB signal at any frequency, so the frequency up converter stage is not required.

* In phasing-type SSB generators, the accuracy of the phase shifters determines the degrees of unwanted sideband suppression.

(ii) It can use the low audio frequencies as modulating signal. (In filter method, this is not possible).

(iii) It is easy to switch from one sideband to the other.

**3. Drawbacks**

(i) The disadvantage is that the design of the 90° phase shifting network for the modulating signal is extremely critical.

(ii) This network has to provide a correct phase shift of 90° at all the modulating frequencies which is practically difficult to achieve.

## 3.32.1. Performance Comparison between the Sideband Suppression Methods

***(RGTU, Bhopal, Sem. Exam., 2006-07) (08 marks)***

Table 3.1 presents the performance comparison between the sideband supperssion methods.

**TABLE 3.1.**

| S.N. | Parameter of comparison | Frequency discrimination method | Phase discrimination Method |
|---|---|---|---|
| (i) | Method to cancel the unwanted sideband. | Using a filter. | By shifting AF and RF signals to BM by 90°. |
| (ii) | Design of 90° shifter at modulating frequency | Not applicable. | Design is critical. |
| (iii) | Possibility of SSB generation at any frequency | Not possible to generate at any frequency. | Possible |
| (iv) | Need of up conversion | Needed | Not needed. |
| (v) | Use of low modulating frequencies. | Not possible | Possible |
| (vi) | Need of linear amplifiers. | Needed | Needed. |
| (vii) | Critical points in system design. | Filter characteristics, its size and weight, cutoff frequency. | Design of 90° phase shifter for modulating frequency. Symmetry of balanced modulators. |

### 3.32.1.1. Why is SSB not used for broadcasting ?

We have seen that there are so many advantages a SSB system has over the DSBFC system. Still it is not used widely in the radio broadcasting applications. There are two reasons for it as under:

(i) As the SSB transmitter and receiver require an excellent frequency stability, a small frequency shift in the system can result in degradation in the quality of the transmitted signal. Thus, it is not possible to transmit a good quality music using the SSB system.

(ii) It is not possible to design a tunable receiver oscillator with very high frequency stability. Now, with the advent of the frequency synthesizers, this has becomes possible. But, such receivers are too expensive.

> **Important Point:** *These are the reasons why SSB is not generally used in the broadcasting applications.*

## 3.32.2. Pilot Carrier SSB System (SSB-RC)

In the practical SSB systems, the carrier is not completely suppressed. Instead, a pilot carrier is transmitted alongwith the desired sideband. The pilot carrier is added to the SSB signal which is to be transmitted. This reinsertion of carrier takes place after the unwanted sideband has been removed. The reinserted carrier is at a very low owner level, 16 to 26 dB below the normal carrier level before suppression. The frequency of the pilot carrier is same as that of the original carrier. This pilot carriers acts as a reference signal to help the demodulation process in the receiver. The receiver can then use the automatic frequency control technique (AFC). The pilot carrier SSB systems are otherwise identical to the SSB systems studied till now. They are widely used in transmarine point to point radio-telephony and mobile communications. This system is also called as Reduced Carrier SSB or SSB-RC system. A simplified SSB-RC system is shown in figure 3.46.

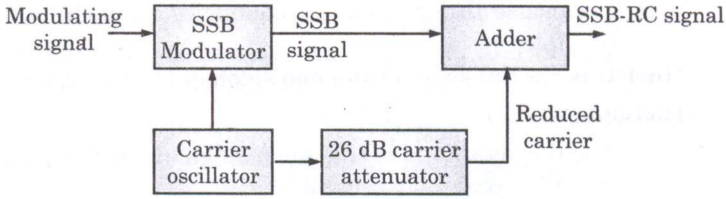


**Fig. 3.46** *Block schematic of an SSB-RC system*

The modulating signal and carrier are applied to an SSB modulator. It can use anyone of the SSB generation methods discussed earlier. The carrier is passed through a 26 dB attenuator to reduce its amplitude. The reduced carrier and SSB signal are added together in the adder block to produce the SSB-RC signal*.

### 3.32.3. Demodulation of SSB Waves

The SSB receivers are normally used for professional or commercial communications. The special requirements of SSB receivers are as follows :

**1. Requirements of SSB receiver**

(*i*) High reliability
(*ii*) Excellent suppression of adjacent signals
(*iii*) High signal to noise ratio
(*iv*) Ability to demodulate SSB

### 3.32.4. Coherent SSB Demodulation** *(RTU, Kota, Sem. Exam; 2002-03) (10 marks)*

**1. Definition and Block Diagram**

The product modulator is a type of coherent SSB demodulator. To recover the modulating signal from the SSB-SC signal, we require a phase coherent or synchronous demodulator. The block diagram of the coherent SSB-SC demodulator is as shown in figure 3.47. The received SSB signal is first multiplied with a locally generated carrier signal. The locally generated carrier should have exactly the same frequency as that of the suppressed carrier. The product modulator multiplies the two signals at its input and the product signal is passed through a low pass filter with a bandwidth equal to $f_m$. At the output of the filter, we get the modulating signal back. Mathematically, this can be proved as follows:

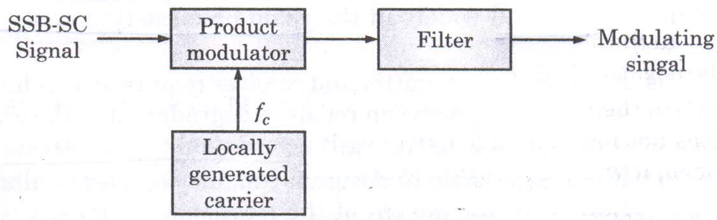


**Fig. 3.47** *Block diagram of coherent SSB demodulator*

**2. Analysis of the Coherent Detector**

Let the SSB wave at the input be given by,

* Precision RC networks are normally used to produce the desired 90° phase shifts.
** A demodulator for SSB is a mixer, such as a balanced modulator, that is called a product detector. The carrier is reinserted in the reciever with a local oscillator.

$$s(t) = \frac{1}{2} V_c \left[x(t)\cos(2\pi f_c t) \pm \hat{x}(t)\sin(2\pi f_c t)\right]$$

The locally generated carrier is $\cos(2\pi f_c t)$

Therefore, output of the product modulator is given by,

$$v(t) = s(t) \,.\, \cos(2\pi f_c t)$$

or $$v(t) = \frac{1}{2} V_c \cos(2\pi f_c t) \,.\, \left[x(t)\cos(2\pi f_c t) \pm x(t)\sin(2\pi f_c t)\right]$$

$$v(t) = \frac{1}{2} V_c\, x(t)\cos(2\pi f_c t)\cos(2\pi f_c t) \pm \frac{V_c}{2}\hat{x}(t)\cos(2\pi f_c t)\sin(2\pi f_c t) \quad \text{... (3.133)}$$

But, $$\cos A \cos B = \frac{1}{2}\{\cos(A+B) + \cos(A-B)\}$$

and $$\cos A \sin B = \frac{1}{2}\{\sin(A+B) + \sin(A-B)\}$$

Therefore, $$v(t) = \frac{1}{4} V_c\, x(t)\left[\cos(4\pi f_c t) + \cos(0)\right] \pm \frac{1}{4} V_c\, \hat{x}(t)\left[\sin(4\pi f_c t) - \sin(0)\right]$$

or $$v(t) = \underbrace{\frac{1}{4} V_c\, x(t)}_{\text{Scaled message signal}} + \underbrace{\frac{1}{4} V_c\left[x(t)\cos(4\pi f_c t) \pm \hat{x}(t)\sin(4\pi f_c t)\right]}_{\text{Unwanted terms}} \quad \text{... (3.134)}$$

When $v(t)$ is passed through the filter, it will allow only the first term to pass through and will reject all other unwanted terms. Thus, at the output of the filter we get the scaled message signal and the coherent SSB demodulation is achieved.

Therefore, $$v_0(t) = \frac{1}{4} V_c\, x(t) \quad \text{... (3.135)}$$

**3. Phase error and frequency error in coherent detection :**

***(BPTU, Orissa, Exam. Exam. 2002-03)***

The coherent detection explained in the preceding section, assumed the ideal operating conditions in which the locally generated carrier is in the perfect synchronization. But, in practice, a phase error $\phi$ may arise in the locally generated carrier wave. The detector output will get modified due to phase error as follows :

$$v_0(t) = \frac{1}{4} V_c\, x(t)\cos\phi \pm \frac{1}{4} V_c\, \hat{x}(t)\sin\phi \quad \text{... (3.136)}$$

In the above expression, the plus sign corresponds to the SSB input signal with only USB whereas the negative sign corresponds to SSB input with only LSB. Due to the presence of the Hilbert transform $\hat{x}(t)$ in the output, the detector output will suffer from the phase distortion. Such a phase distortion does not have serious effects with the voice communication. But in the transmission of music and video, it will have untolerable effects.

## 3.33 VESTIGIAL SIDEBAND TRANSMISSION (VSB)*

***(GTU, Gujrat, Kota Sem. Exam; 2007-08)***

The stringent frequency-response requirements on the sideband filter in SSB-SC system can be relaxed by allowing a part of the unwanted sideband (called as Vestige) to appear in the output of

* Discuss how the VSB modulation is used in commerical TV signal. Discuss its merits and demerits.