3.18 INTRODUCTION TO DOUBLE SIDEBAND SUPPRESSED CARRIER (DSB-SC) SYSTEM*

(PTU, Jalandhar, Sem. Exam; 2006-07)

1. Definition and Mathematical Expression

The equation of AM wave in its simplest form *i.e.*, single-tone modulation, is expressed as

$$s(t) = A\cos\omega_c t + A.\frac{m_a}{2}\cos(\omega_c + \omega_m)t + A.\frac{m_a}{2}\cos(\omega_c - \omega_m)t \quad ...(3.71)$$

From this equation, it is obvious that the carrier component in AM wave remains constant in amplitude and frequency. This means that the carrier of amplitude modulated wave does not convey any information. In power calculation of AM signal, it has been observed that for single-tone sinusoidal modulation, the ratio of the total power to the carrier power is $\left(1 + \frac{m_a^2}{2}\right)$, m_a being the modulation index. Thus for 100% modulation about 67% of the total power is required for transmitting the carrier which does not contain any information. Hence, if the carrier is suppressed, only the sidebands remain and in this way a saving of two-third power may be achieved at 100% modulation, This type of suppression of carrier does not affect the baseband signal in any way. The resulting signal obtained by suppressing the carrier from the modulated wave is called **Double sideband suppressed carrier (DSB-SC) system.**

Thus, in a Double-sideband suppressed carrier modulation there is no carrier signal only sidebands are present.

We know that the frequency-shifting property of Fourier transform is given as

If $\quad x(t) \longleftrightarrow X(\omega)$

then $\quad e^{j\omega_c t}\, x(t) \longleftrightarrow X(\omega - \omega_c) \quad ...(3.72)$

This property states that if a signal $x(t)$ is multiplied by $e^{j\omega_c t}$ in time-domain then its spectrum $X(\omega)$ in frequency-domain is shifted by an amount ω_c.

Similarly,

$$e^{-j\omega_c t}\, x(t) \longleftrightarrow X(\omega + \omega_c) \quad ...(3.73)$$

But, since $e^{j\omega_c t}$ is not a real function and cannot be generated practically, therefore, frequency shifting in practice is achieved by multiplying $x(t)$ by a sinusoid such as $\cos \omega_c t$.

Hence, $\quad x(t)\cos\omega_c t = x(t)\cdot\frac{1}{2}(e^{j\omega_c t} + e^{-j\omega_c t}) = \frac{1}{2}x(t)\,e^{j\omega_c t} + \frac{1}{2}x(t)\,e^{-j\omega_c t} \quad ...(3.74)$

Using frequency-shifting property in equation (3.71), we get

$$x(t)\cos\omega_c t \longleftrightarrow \frac{1}{2}[X(\omega - \omega_c) + X(\omega + \omega_c)] \quad ...(3.75)$$

This means that the multiplication of a signal $x(t)$ by a sinusoid of frequency ω_c shifts the spectrum $X(\omega)$ by $\pm\omega_c$. Now, if $x(t)$ is taken as modulating or baseband signal and $\cos\omega_c t$ is taken as carrier signal, then $x(t)\cos\omega_c t$ represents the modulated signal. Further, the Fourier transform of this modulated signal is given by the equation (3.75). This equation shows that the spectrum of modulated signal contains only shifted spectrum of signal and there is no carrier components. However, we know that the modulated signal, which contains

DO YOU KNOW?

The DSB signals are used in FM and TV broadcasting to transmit two-channel stereo signals. They are also used in some types of phase-shift keying which is used for transmitting binary data.

* An AM signal with suppressed carrier is called a double-sideband (DSB) signal.

no carrier but two sidebands is called **Double-Sideband Suppressed Carrier (DSB-SC) modulation***.

This means that the term $x(t) \cos \omega_c t$ represents a DSB-SC signal.

2. Block Diagram

Therefore, a DSB-SC signal is obtained by simply multiplying modulating signal $x(t)$ with carrier signal $\cos w_c t$. This is achieved by a product modulator. The block diagram of a product modulator is shown in figure 3.20. Figure 3.21 shows the modulating signal and its Fourier transform, carrier signal and its Fourier transform and DSB-SC signal and its frequency spectrum.

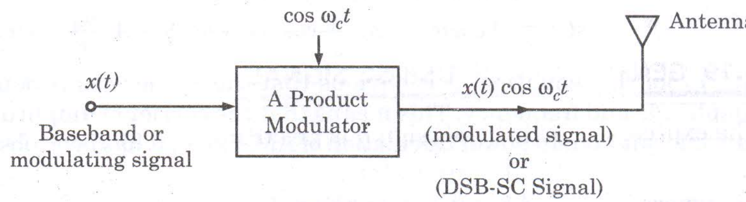


Fig. 3.20.

3. Few points

(*i*) It is obvious from the figure 3.21, that the DSB-SC signal exhibits phase-reversal at zero crossings, *i.e.*, whenever the baseband signal $x(t)$ crosses zero. Because of this, the envelope of a DSB-SC modulated signal is different from the message signal. This is unlike the case of an AM wave.

(*ii*) From figure 3.21, it is also clear that the impulses at $\pm w_c$ are missing which means that the carrier term is suppressed in the spectrum and only two sideband terms, USB and LSB are left. Therefore, it is called double sideband suppressed carrier (DSB-SC) system.

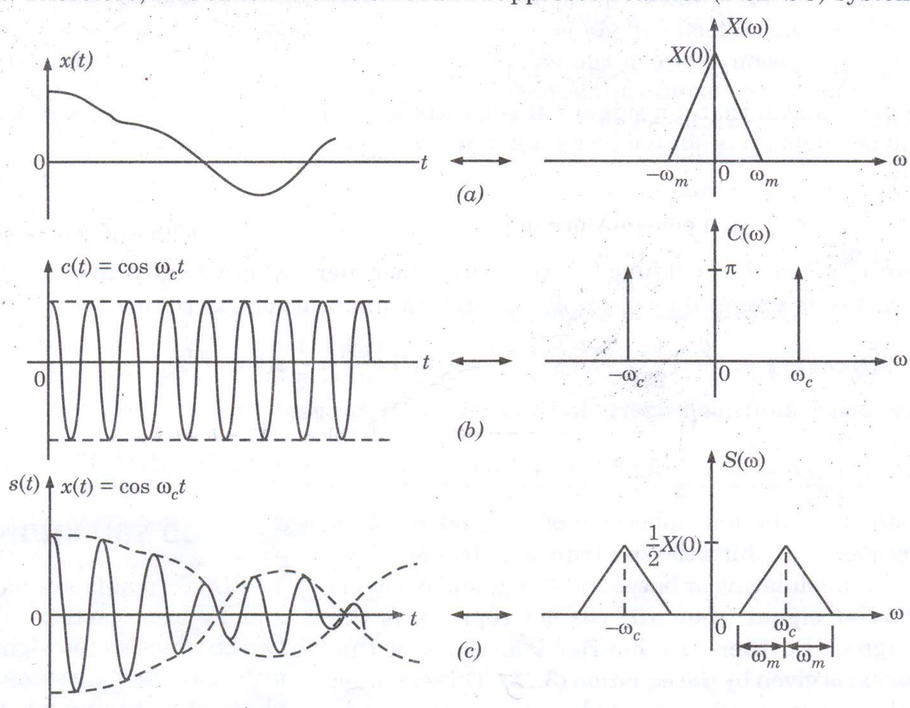


Fig. 3.21

* In an AM signal, the carrier contains no information. Any transmitted information lies solely in the sideband. For that reason, the carrier may be suppressed and not transmitted.

(*iii*) In figure 3.21, considering only positive side, the upperside band frequency is $\omega_c + \omega_m$ whereas the lowerside band frequency is $\omega_c - \omega_m$. The difference of these two is equal to the **transmission bandwidth of a DSB-SC signal,** *i.e.*

Bandwidth $B = (\omega_c + \omega_m) - (\omega_c - \omega_m) = 2\,\omega_m$

It is obvious that the bandwidth of DSB-SC modulation is same as that of general AM wave.

3.19 GENERATION OF DSB-SC SIGNAL

The expression for DSB-SC signal is given as

$$s(t) = x(t)\cos\omega_c t$$

where, $x(t)$ = Baseband signal

$\cos\omega_c t$ = Carrier signal

From this expression, we see that a DSB-SC signal is basically the product of the modulating or baseband signal and the carrier signal. Unfortunately, a single electronic device cannot generate a DSB-SC signal. As discussed earlier, a circuit to achieve the generation of a DSB-SC signal is called a product modulator. In this section, we shall discuss two types of product modulator, namely the **Balanced Modulator** and the **Ring Modulator.**

3.19.1. Modulation Using Nonlinear Devices (Balanced Modulator)*

The modulation can also be achieved by using the nonlinear devices. Figure 3.22 (*a*) shows the typical non linear device characteristics. A semiconductor diode is a good example of a non linear device. The relation between the voltage across a device (v) and the device current (i) is nonlinear as shown in figure 3.19(*a*) and it can be mathematically expressed as

$$i = av + b\,v^2$$

where a and b are constants.

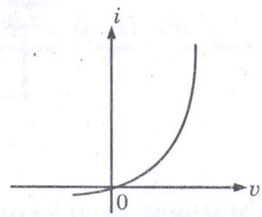


Fig. 3.22 (*a*) *Nonlinear characteristics of a device*

Figure 3.22 (*b*) shows a possible arrangement for the use of nonlinear elements for DSB-SC modulator.

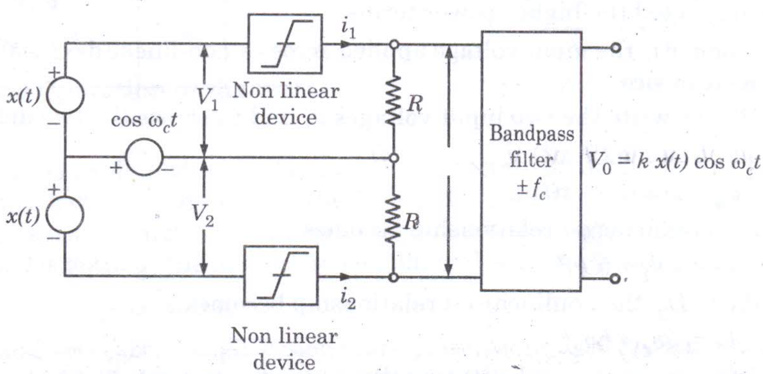


Fig. 3.22 (*b*) *DSB-SC modulator using nonlinear device*

Such an arrangement is called as **balanced modulator.**

* Balanced modulators are AM circuits that cancel or suppress the carrier but generate a DSB output signal that contains the upper (sum) and lower (difference) sideband frequencies.

3.20 THE BALANCED MODULATOR USING DIODES

(GGSIPU, Delhi, Semester, Exam., 2004-05)

1. Definition

We know that a non-linear resistance or a non-linear device may be used to produce Amplitude Modulation, *i.e.*, one carrier and two sidebands. However, a DSB-SC signal contains only two sidebands. Thus, if two non-linear devices such as diodes, transistors etc. are connected in a balanced mode so as to suppress the carriers of each other, then only sidebands are left, *i.e.*, a DSB-SC signal is generated. Therefore, a balanced modulator may be defined as a circuit in which two non-linear devices are connected in a balanced mode to produce a DSB-SC signal. In the following section, we shall discuss a balanced modulator circuit using diodes.

2. Circuit Diagram

Figure 3.23 shows a balanced modulator circuit using two diodes. A modulating signal $x(t)$ is applied to the two diodes through a centre-tapped transformer with the carrier signal cos $w_c t$.

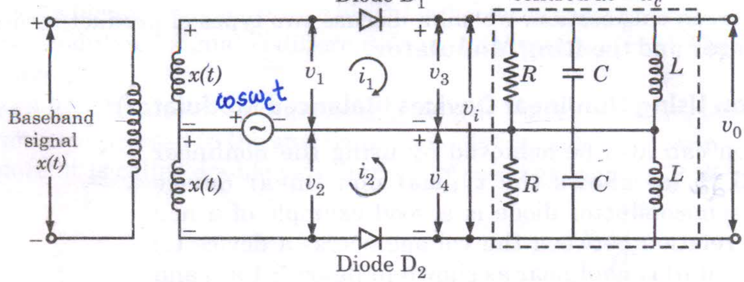


Fig. 3.23. *Balanced Modulator using Diodes*

3. Mathematical Expression

A non-linear v-i relationship is given as

$$i = av + bv^2 \quad ...(3.76)$$

Here, we have neglected the higher power terms.

In above expression, v is the input voltage applied across a non-linear device and i is the current through the non-linear device.

From figure 3.23, we write the two input voltages v_1 and v_2 across the two diodes as

$$v_1 = \cos \omega_c t + x(t) \quad ...(3.77)$$

$$v_2 = \cos \omega_c t - x(t) \quad ...(3.78)$$

For diode D_1, the nonlinear v-i relationship becomes

$$i_1 = av_1 + b\, v_1^2 \quad ...(3.79)$$

Similarly, for diode D_2, the nonlinear v-i relationship becomes

$$i_2 = av_2 + bv_2^2 \quad ...(3.80)$$

In the expression of current i_1, substituting the value of v_1, we get

$$i_1 = a\,[\cos \omega_c t + x(t)] + b\,[\cos \omega_c t + x(t)]^2$$

or $$i_1 = a \cos \omega_c t + ax(t) + b\,[\cos^2 \omega_c t + x^2(t) + 2x(t) \cos \omega_c t]$$

or $$i_1 = a \cos \omega_c t + ax(t) + b \cos^2 \omega_c t + bx^2(t) + 2bx(t) \cos \omega_c t \quad ...(3.81)$$

Similarly, in the expression of current i_2, substituting the value of v_2, we get

$$i_2 = a\,[\cos \omega_c t - x(t)] + b[\cos \omega_c t - x(t)]^2$$

or $$i_2 = a\cos\omega_c t - ax(t) + b\cos^2\omega_c t + bx^2(t) - 2bx(t)\cos\omega_c t \quad ...(3.82)$$

Due to currents i_1 and i_2, the net voltage v_i at the input of bandpass filter is expressed as

$$v_i = v_3 - v_4 \quad ...(3.83)$$

But from figure, we have

$$v_3 = i_1 R$$

and $$v_4 = i_2 R$$

Therefore, $$v_i = i_1 R - i_2 R$$

or $$v_i = R(i_1 - i_2)$$

In above equations, substituting the values of i_1 and i_2 from equations (3.81) and (3.82) we get

$$v_i = R[2ax(t) + 4bx(t)\cos\omega_c t]$$

or $$v_i = 2R[ax(t) + 2bx(t)\cos\omega_c t]$$

This voltage v_i is the input to the bandpass filter centered around $\pm w_c$.

A bandpass filter is that type of filter which allows to pass a band of frequencies. Here, since the bandpass filter is centred around $\pm\omega_c$, it will pass a narrow band of frequencies centred at $\pm\omega_c$ with a small bandwidth of $2\omega_m$ to preserve the sidebands.

Therefore, the output of BPF centred around $\pm w_c$ is given by

$$v_0 = 4b\,Rx(t)\cos\omega_c t = K\,x(t)\cos\omega_c t$$

which is the expression for a DSB-SC signal.

3.21 RING MODULATOR OR CHOPPER MODULATOR FOR THE DSB-SC GENERATION*

Figure 3.24 shows the circuit diagram of a diode ring modulator. It consists of four diodes, an audio frequency transformer T_1 and an RF transformer T_2. The carrier signal is assumed to be a square wave with frequency f_c and it is connected between the centre taps of the two transformers. The DSB-SC output is obtained at the secondary of the RF transformer T_2.

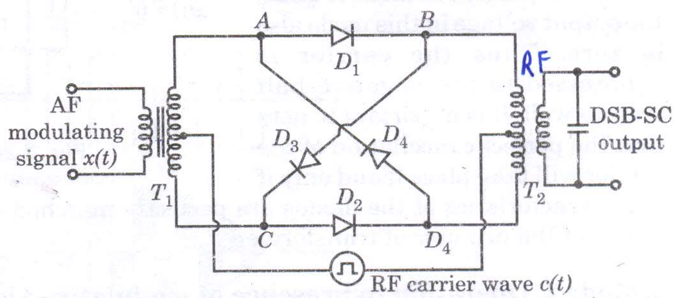


Fig. 3.24. *A diode ring modulator*

3.21.1. Working Operation of the Circuit

The operation is explained with the assumptions that the diodes act as perfect switches and that they are switched ON and OFF by the RF carrier signal. This is because the amplitude and frequency of the carrier is higher than that of the modulating signal. The operation can be divided into different modes without the modulating signal and with modulating signal as follows :

1. Mode 1: Carrier Suppression

To understand how carrier suppression takes place, let us assume that the modulating signal is absent and only the carrier signal is applied. Hence, $x(t) = 0$.

(*i*) Operation in the positive half cycle of carrier

In this mode, let us assume that the modulating signal is zero, and only the carrier signal is

* A popular balanced modulator is the lattice modulator that uses a diode bridge circuit as a switch. The carrier turns the diodes off and on, letting segments of the modulating signal through to produce a DSB output signal. A carrier suppression of 40 dB is possible.

applied. The equivalent circuit for this mode of operation is as shown in figure 3.25. As shown, the diodes D_1 and D_2 are forward biased. Diodes D_3 and D_4 are reverse biased.

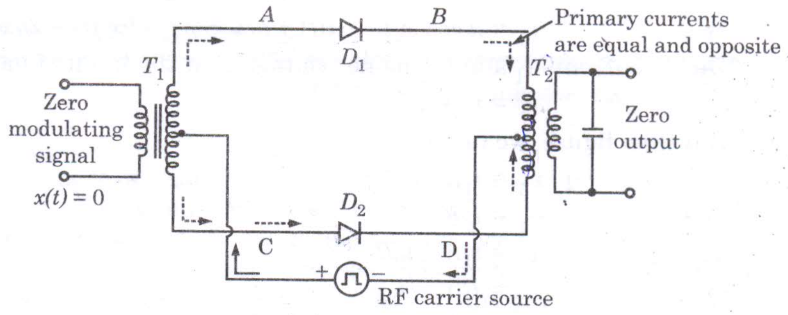


Fig. 3.25 *Equivalent circuit in mode I*

Let us observe the directions of currents flowing through the primary windings of output transformer T_2, they are equal and opposite to each other. Therefore, the magnetic fields produced by these currents are equal and opposite and cancel each other. Hence, the induced voltage in secondary winding is zero. Thus, the carrier is supported in the positive half cycle.

(ii) Operation in negative half cycle of the carrier

In this mode also, let us assume that the modulating signal is zero. In the negative half cycle of the carrier, the diodes D_3 and D_4 are forward biased and the diodes D_1 and D_2 are reverse biased. In figure 3.26, the currents flowing in the upper and the lower halves of the primary winding of T_2 are again equal and in opposite directions. This is going to cancel the magnetic fields as explained in mode *I*. Thus, the output voltage in this mode also is zero. Thus the carrier is suppressed in the negative half cycle as well. It is important to note that the perfect cancellation of the carrier will take place if and only if the characteristics of the diodes are perfectly matched and the centre tap is placed exactly at the centre of the primary of transformer T_2.

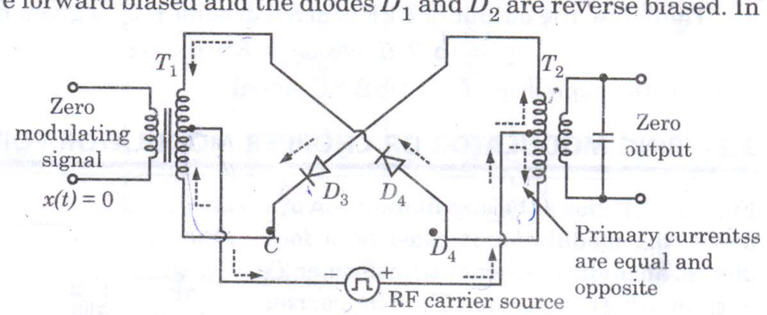


Fig. 3.26 *Equivalent circuit in mode II*

2. Mode 2. Operation in presence of modulating signal

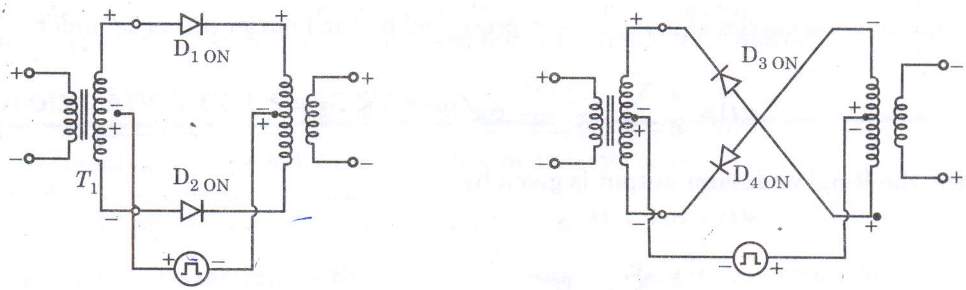
Now, let us discuss the operation when RF carrier and the modulating signal both are applied.

(i) Operation in the positive half cycle of modulating signal

(*i*) As we apply the low frequency modulating signal through the input audio transformer T_1, there are many cycles of the carrier signal, in the positive half cycle of the modulating signal.

(*ii*) In the positive half cycle of the carrier, D_1 and D_2 are on and secondary of T_1 is applied as it is across the primary of T_2. Hence, during the positive half cycle of carrier, the output of T_2 is positive as shown in figure 3.27 (*a*).

(*iii*) In the negative half cycle of the carrier, D_3 and D_4 are turned on and the secondary of T_1 is applied in a reversed manner across the primary of T_2 as shown in equivalent circuit of figure 3.27 (*b*). Thus, the primary voltage of T_2 is negative and output voltage also becomes negative.



(*a*) Equivalent circuit in the positive half cycle of modulating signal with carrier positive

(*b*) Equivalent circuit in the negative half cycle of modulating signal with carrier negative

Fig. 3.27

(*ii*) Operation in the negative half cycle of modulating signal

When modulating signal reverses the polarities, the operation of the circuit is same as that in the positive half cycle discussed earlier. Now, the only difference is that the diode pair D_3 D_4 will produce a positive output voltage whereas D_1 D_2 will produce a negative output voltage as shown in the waveforms of figure 3.28.

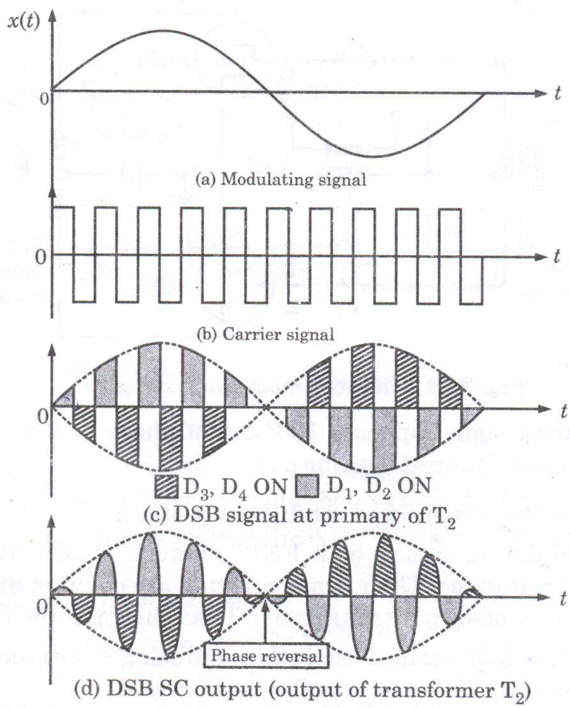


Fig. 3.28 *Waveforms of the lattice type balanced modulator*

3.21.2. Analysis of Ring Modulator

From the discussion till now, it is clear that in the positive half cycle of the carrier, the message signal $x(t)$ is multiplied by + 1 and in the negative half cycle of the carrier, $x(t)$ is multiplied by – 1. Thus, the ring modulator is an ideal form of product modulator and hence it produces the required DSB-SC output.

The square-wave carrier signal can be represented by the Fourier series as under :

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c t(2n-1)\right]$$

Hence, the Ring-Modulator output is given by

$$s(t) = x(t) \cdot c(t)$$

$$= \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c t(2n-1)\right] x(t)$$

It may be observed that there is no output from the modulator at carrier frequency. Therefore, the carrier is entirely eliminated.

3.22 BALANCED MODULATOR USING FETS

The balanced modulator using FETs is as shown in figure 3.29. The carrier voltage is applied in phase to the two gates via the transformers T_2 and T_1.

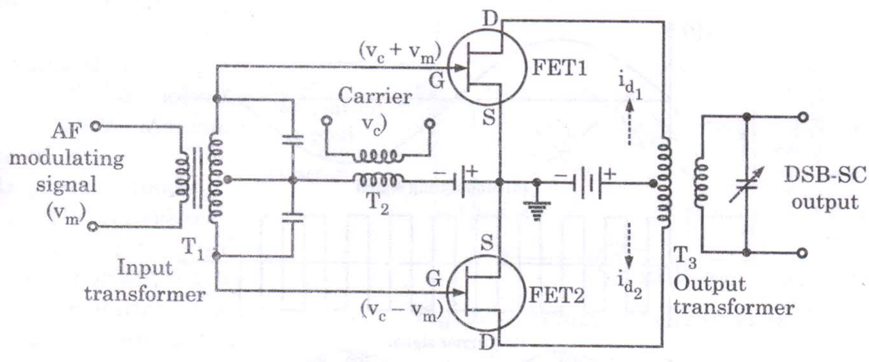


Fig. 3.29 *Balanced modulator using FETs*

However, the modulating signal appears 180° out of phase at the gates. This is because the input transformer T_1 is a centre tapped transformer.

(*i*) Mode 1 (Operation in the absence of modulating signal)

In the absence of modulating signal, both FETs conduct simultaneously due to the inphase carrier voltage applied to their gates. Their drain currents are equal in magnitudes but opposite in direction through the primary of output transformer T_3 as shown in the figure 3.29.

Due to this their negative fields cancel each other, inducing a zero secondary voltage. Thus, the output of transformer T_3 is zero. The carrier is thus suppressed.

(*ii*) Mode II (Carrier and modulating signal both present)

When the modulating signal is applied, the drain currents of the two FETs flow due to the combined effect of carrier and the modulating signal. The FET currents due to carrier are equal and opposite and hence cancel each other. However, FET currents due to the modulating signal are equal but not opposite so they do not cancel out. This is because the modulating signal is applied 180° out of phase to the two FETs.

Hence, at the output of the circuit, we get a DSB-SC signal. For the 100% suppression of the carrier, both the FETs must have the identical characteristics *i.e.*, it should be a matched pair and the transformer centre taps must be exactly at the centre of the windings. Practically this is not possible hence carrier will be heavily suppressed but not completely removed*.

3.23 DEMODULATION OF DSB-SC SIGNALS

The DSB-SC signal may be demodulated by following two methods:

(*i*) Synchronous detection method

(*ii*) Using envelope detector after carrier reinsertion.

3.23.1. Synchronous Detection Method *(G.G.U.T., Bhilai, Sem. Exam; 2005-04) (06 marks)*

1. Definition

We know that the DSB-SC system is used at the transmitter end to shift the modulating signal (having maximum frequency ω_m) to a higher carrier frequency $\pm\ \omega_c$. Now, this modulated (DSB-SC) signal is transmitted from the transmitter and it reaches the receiver through a transmission medium. At the receiver end, the original modulating signal $x(t)$ is recovered from the modulated (DSB-SC) signal. This can be achieved by simply retranslating the baseband or modulating signal from a higher spectrum, centered at $\pm\omega_c$, to the original spectrum. This process of retranslation is called **demodulation or detection**. Hence, the original or baseband signal is recovered from the modulated signal by the detection process.

2. Block Diagram

A method of DSB-SC detection is known as **synchronous detection.** Figure 3.30 shows the block diagram of synchronous detection method.

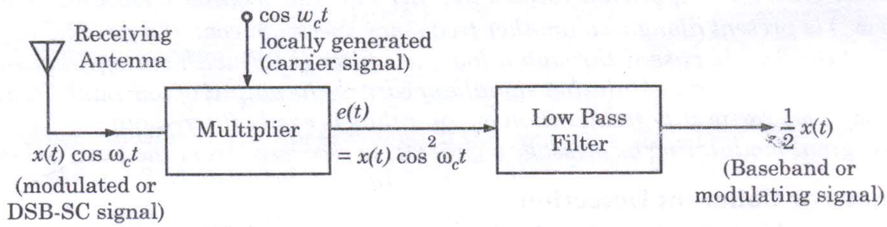


Fig. 3.30 *Synchronous detection method*

3. Working Principle

In synchronous detection method, the received modulated or DSB-SC signal is first multiplied with a locally generated carrier signal cos $\omega_c t$ and then passed through a low-pass filer (LPF). At the output of a low-pass filter (LPF), the original modulating signal is recovered.

Mathematically,

$$e(t) = \underbrace{x(t) \cos \omega_c t}_{\text{DSB-SC signal}} \cdot \underbrace{\cos \omega_c t}_{\text{locally generated carrier signal}} \quad ...(3.85)$$

or

$$e(t) = x(t) \cos^2 \omega_c t = \frac{1}{2} x(t)\left[2\cos^2 \omega_c t\right]$$

or

$$e(t) = \frac{1}{2} x(t)[1 + \cos 2\omega_c t] = \frac{1}{2} x(t) + \frac{1}{2} x(t) \cos 2\,\omega_c t \quad ...(3.86)$$

Now, it may be observed that when multiplied signal $e(t)$ is passed through a low-pass filter

* Another widely used balanced modulator is an integrated circuit (IC) using differential amplifiers as switches to switch the modulating signal at the carrier frequency. A popular device is the 1496 or 1596. Carrier suppresion can be as high as 50 to 65 dB.

(LPF), then the term $\frac{1}{2}x(t)\cos 2\omega_c t$, centred at $\pm 2\omega_c$ is suppressed by low-pass filter and thus at the output of low-pass filter, the original modulating signal $\frac{1}{2}x(t)$ is obtained.

We may get the frequency-spectrum of multiplied signal $e(t)$ using Fourier Transform as

$$x(t)\cos^2\omega_c t \longleftrightarrow \frac{1}{2}X(\omega) + \frac{1}{4}[X(\omega + 2\omega_c) + X(\omega\ \ 2\omega_c)] \quad ...(3.87)$$

4. Frequency Spectrum

Figure 3.31 shows this frequency spectrum

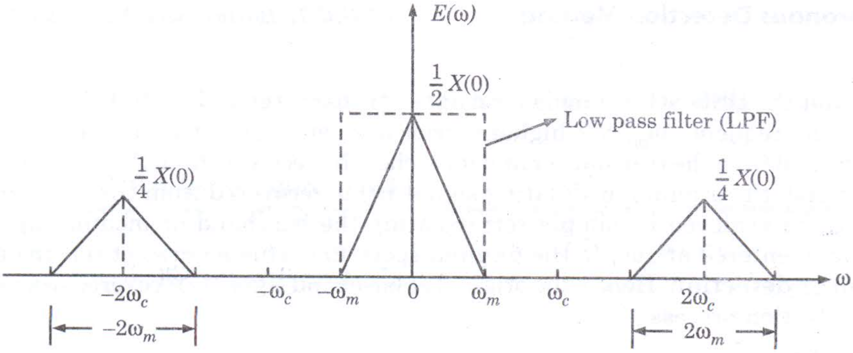


Fig. 3.31 *Frequency spectrum of $x(t)\cos^2\omega_c t$*

Important Note: *The spectrum reveals the fact that the original baseband or modulating signal (0 to ω_m) is present alongwith another frequency spectrum centered around $\pm\omega_c$ when this signal i.e., $x(t)\cos 2\omega_c t$ is passed through a low pass filter (LPF) with cut-off frequency ω_m, the original baseband signal or modulating signal appears at the output of low pass-filter. It may be noted $\omega_c >> \omega_m$ and $2\omega_c$ is still greater than ω_m and thus is easily filtered out. Hence, the original modulating or message signal x(t) is recovered from the DSB-SC signal.*

5. Synchronous or Coherent Detection

We have observed in last article that the detection process for DSB-SC requires a local oscillator at the receiver end. The frequency and phase of the locally generated carrier signal and the carrier signal at the transmitter carrier must be identical. This means that the local oscillator signal must be exactly coherent or synchronized with the carrier signal at the transmitter, both in frequency and phase, otherwise the detected signal would get distorted. Therefore, this method of recovery is called **synchronous detection** or **coherent detection**. Thus the demerits of the synchronous detection is that if requires an additional system at the receiver to ensure that the locally generated carrier is synchronized with the transmitter carrier making the receiver complex and costly.

3.23.2. Envelope Detection after Suitable Carrier Re-insertion

1. Definition

The other possible method of demodulating DSB-SC signal is by inserting a carrier generated at the receiver end with the help of a local oscillator. However, the phase and the frequency of the re-inserted carrier must be properly synchronized with those at the transmitter end in order to avoid distortion. We know that if we insert a sufficient carrier of same frequency and phase to DSB-SC signal, it converts DSB-SC signal into a conventional AM wave. Now, this AM wave is demodulated by an envelope detector. However, phase and frequency errors will result in similar type of distortion are obtained in coherent detection.

2. Mathematical Expressions

Let us consider that the received DSB-SC signal is expressed by

$$s(t) = c(t) \cdot x(t)$$

or
$$s(t) = A \cos (2\pi f_c t)\, x(t) \quad ...(3.88)$$

Let us assume $A = 1$ in equation (3.88). Then equation (3.88) can be written as

$$s(t) = \cos (2\pi f_c t)\, x(t) \quad ...(3.89)$$

The inserted carrier at the receiver will be

$$c'(t) = A \cos (2\pi f_c t + \phi) \quad ...(3.89a)$$

where ϕ = amount of phase discrepancy

Then the resulting signal will be

$r(t) = s(t)$ + inserted carrier at the receiver

or
$$r(t) = s(t) + c'(t) \quad ...(3.90)$$

substituting equations (3.89) and (3.89*a*) in equation (3.90), we get

$$r(t) = s(t) + c'(t) = \cos (2\pi f_c t) x(t) + A \cos (2\pi f_c t + \phi)$$

or
$$r(t) = x(t) \cos (2\pi f_c t) + A \cos (2\pi f_c t) \cos \phi - A \sin (2\pi f_c t) \sin \phi$$

or
$$r(t) = [x(t) + A \cos \phi] \cos (2\pi f_c t) - (A \sin \phi) \sin (2\pi f_c t)$$

or
$$r(t) = e(t) \cos[(2\pi f_c t) + \theta(t)] \quad ...(3.91)$$

where
$$e(t) = \sqrt{[A + x(t)]^2 - 2A\, x(t)[1 - \cos\phi]}$$

and
$$\theta(t) = \tan^{-1}\left[\frac{A \sin\phi}{x(t) + A\cos\phi}\right]$$

Now, from the expression

$$r(t) = e(t) \{\cos [(2\pi f_c t) + \theta(t)]\}$$

it may be observed that $e(t)$ is the envelope of the resulting signal $r(t)$.

Also, if we take $\phi = 0$, then envelope will be given by

$$e(t) = A + x(t)$$

Hence, modulating signal $x(t)$ can be recovered from $r(t)$ using an envelope detector since the $r(t)$ is basically a conventional AM wave given by

$$r(t) = [A + x(t)] \cos 2\pi f_c t \quad ...(3.92)$$

This is however possible only when $[A + x(t)] > 0$ for all values of t.

It is possible only when the modulation index m is less than unity.

If $\phi \neq 0$, then the phase error exists between the two carriers. It is given as

$$e(t) = A\left[1 + \frac{2x(t)}{A} \cos\phi + \left\{\frac{x(t)}{A}\right\}^2\right]^{\frac{1}{2}} \quad ...(3.89)$$

If $A >> |x(t)|$, then, we have

$$e(t) \cong A + x(t) \cos \phi \quad ...(3.93)$$

The desired signal output will thus be $x(t) \cos \phi$. If $\phi = 0$ and there is a difference in frequency Δf between the two oscillators, then the envelope of the resulting signal $r(t)$ will be given by

$$e(t) = A + x(t) \cos [2\pi \Delta f t] \qquad \text{for } A >> |x(t)| \quad ...(3.94)$$

Important Point: *The envelope can be detected by an envelope detector but here distortion will be identical to coherent detection.*

null effect since the signal is zero when the local carrier is in phase quadrature with the transmitted carrier signal.

(*iii*) When there is only the frequency error, *i.e.*

$$\Delta\omega \neq 0 \quad \text{and} \quad \phi = 0$$

In this case, equation (3.96) gives

$$e_0(t) = \frac{1}{2} x(t) \cos(\Delta\omega)\, t$$

Here, the multiplying factor cos ($\Delta\omega$) t is time-dependent and produces distortion in the detected output signal. The error $\Delta\omega$ is usually small and thus a message signal $x(t)$ is multiplied by a slow varying sinusoidal signal. This is a more serious distortion. Therefore a frequency error must be avoided

(*iv*) When both errors are non-zero, *i.e.*

$$\Delta\omega \neq 0 \quad \text{and} \quad \phi \neq 0$$

In this case equation (3.96) itself provides the detected output signal. Also, in this case, the constant phase error provides attenuation and the frequency error produces distortion in the detected output signal. Hence, we get an attenuated and distorted output signal at the receiver end.

3.26 CARRIER ACQUISITION IN DSB-SC SYSTEM OR SYNCHRONIZATION TECHNIQUES IN DSB-SC SYSTEM

As discussed in the last subsection, the phase and the frequency of the locally generated carrier signal in synchronous detector is very critical. Precision phase and frequency control of the local carrier requires an expensive and a complex circuitry at the receiver end. Some important synchronization techniques are given as under:

1. Pilot Carrier

A small amount of carrier signal known as **pilot carrier** is transmitted alongwith the modulated signal from the transmitter. This small amount of carrier signal is called **pilot carrier**. This pilot carrier, separated at the receiver by an appropriate filter, is amplified, and is used to phase lock the locally generated carrier signal at the receiver. The phase locking provides synchronization. This system, where a weak carrier is transmitted alongwith the DSB-SC signal is also referred to as **partially suppressed carrier system** as the carrier is not totally suppressed. The process in which a large carrier is transmitted alongwith DSB-SC signal is known as amplitude modulation. This has been already discussed. The large carrier simplifies the reception system. The DSB-SC with partially suppressed carrier is equivalent to an over modulated AM signal.

2. Costa's Receiver

This system used for synchronous detection of DSB-SC signal has been shown in figure 3.33.

This system has two synchronous detectors-one detector is fed with a locally generated carrier signal which is in phase with the transmitted carrier signal. This detector circuit is called inphase coherent detector or I-channel. The other synchronous detector employs a local carrier which is in phase quadrature with the transmitted carrier signal and is called Quadrature phase coherent detector or *Q*-channel. On combining, the two detectors constitute a negative feedback system which synchronizes the local carrier signal with the transmitted carrier signal.

(*i*) Block Diagram

Figure 3.33 shows a costa's receiver.

(*ii*) Operating Principle

To start with, let us assume that the local carrier signal is synchronized with the transmitted carrier signal and $\phi = 0$. As shown in figure 3.33 the output of the I-channel is the desired modulating signal (since cos ϕ = 1), but the output of the Q-channel is zero (since sin ϕ = 0) because of the quadrature null effect. Now, assuming that the local oscillator frequency drifts slightly *i.e.*, ϕ is a very small non-zero quantity, I-channel output will be almost unchanged, but Q-channel output now is not a zero, rather some signal would appear at its output and is proportional to sin ϕ. Thus, the output of the Q-channel,

(*i*) is proportional to ϕ (since $\sin\phi = \phi$ for small ϕ)

(*ii*) would have a polarity same as the I-channel for one direction of phase shift in local oscillator, whereas, the polarity would be opposite to I-channel for the other direction of phase shift.

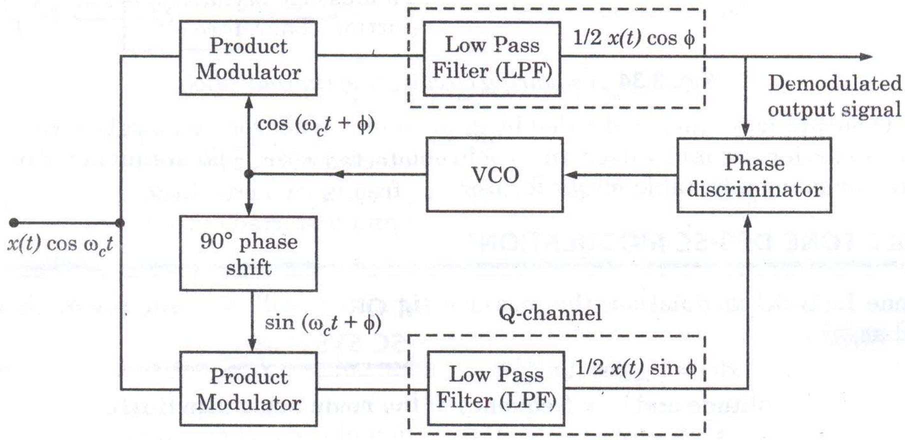


Fig. 3.33 *A Costa's receiver*

The phase discriminator provides a dc control signal which may be used to correct local oscillator phase error. The local oscillator is a voltage controlled oscillator (VCO). Its frequency may be adjusted by an error control d.c. signal.

(*iii*) Limitation

The costa's receiver cases phase control when there is no modulation *i.e.*, $x(t) = 0$. The phase control reestablishes itself on the reappearance of modulation. However, the reestablishment is so fast that distortion is not perceptible in case of voice communication.

3. Squaring Loop

In this method, the received signal is squared by a squaring circuit as shown in figure 3.34. The output of the squarer will be given as

$$[A.\, x(t) \cos \omega_c t]^2 = A^2\, x^2(t) \cos^2 \omega_c t$$

For simplicity let us assume that $x(t)$ is a single tone sinusoidal denoted as $\cos \omega_m t$ *i.e.*,

$$x(t) = \cos \omega_m t$$

then the output of the squarer becomes

$$[A \cos \omega_c t.\, \cos \omega_m t]^2 = A^2 \cos^2 \omega_m t \cos^2 \omega_c t$$

$$= \frac{A^2}{4}(1+\cos 2\omega_m)\,(1+\cos 2\omega_c t)$$

$$= \frac{A^2}{4}[1+\cos 2\omega_m t+\cos 2\,\omega_c t+\cos 2\omega_c t\, \cos 2\omega_m t] \quad ...(3.97)$$

The term $\cos 2\,\omega_c t$ can be obtained by using a narrowband filter centred at $\pm 2\omega_c$. This frequency $\pm 2\omega_c$ is kept constant by tracking through a phase locked loop (PLL). The PLL uses negative feedback technique to provide a constant frequency signal, $\cos 2\,\omega_c t$.

Any drift in frequency is corrected by an error signals $e(t)$, generated at the output of the lowpass filter of PLL as depicted in figure 3.34. PLL will be discussed later on.

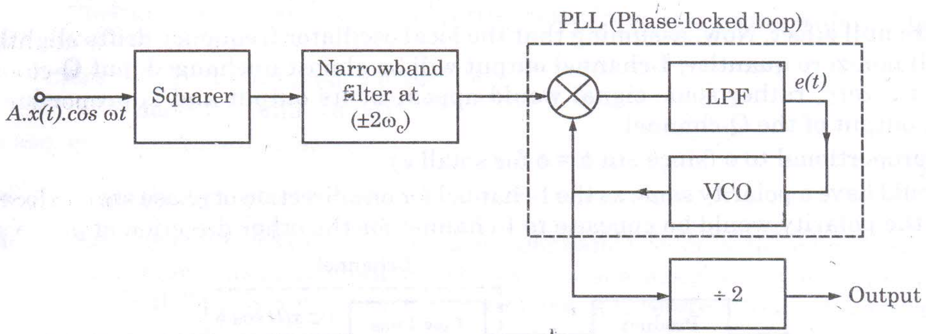


Fig. 3.34 *A squaring circuit for synchronization*

The VCO output is frequency divided by 2, to yield a synchronized local carrier of frequency ω_c. This local carrier signal is used in synchronous detector. The frequency division can be accomplished by using a bistable multivibrator.

3.27 SINGLE TONE DSB-SC MODULATION*

In single tone DSB-SC modulation, the message signal $x(t)$ will be a sinusoidal signal and it is represented as,

$$x(t) = V_m \cos(2\pi f_m t) \quad \text{... (3.98)}$$

where V_m = amplitude and f_m = frequency of the modulating signal $x(t)$.

3.27.1. Time Domain Description

The DSB-SC modulated wave is given by,

$$s(t) = x(t) \cdot c(t) = V_m \cos(2\pi f_m t) \cdot V_c \cos(2\pi f_c t)$$

or

$$s(t) = V_m V_c \cos(2\pi f_c t) \cdot \cos(2\pi f_m t) \quad \text{...(3.99)}$$

But, we know that $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

Therefore, $\cos(2\pi f_c t)\cos(2\pi f_m t) = \frac{1}{2}[\cos 2\pi(f_c + f_m)t + \cos 2\pi(f_c - f_m)t]$ ... (3.100)

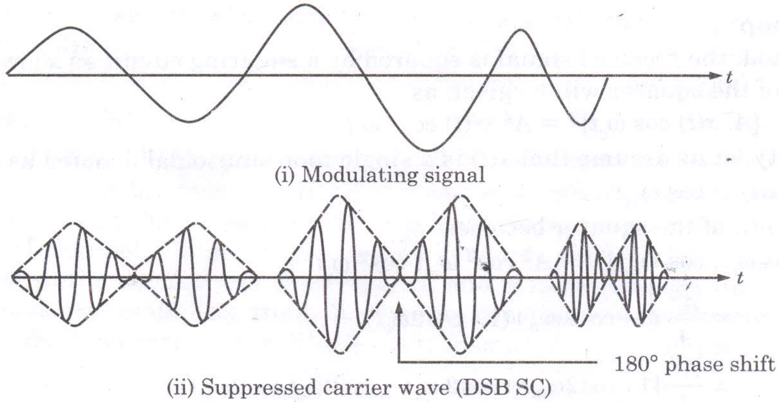


Fig. 3.35 *Time domain description of signal tone DSB-SC modulation*

* Since the same transmitted information is contained in both upper and lower sidebands, one is redundant. Full information can be transmitted using only one sideband.

3.27.3. Power Saving*

Due to suppression of carrier, a lot of power saving takes place in DSB-SC. At 100% modulation, $m = 1$, the percent power saving is given by $(P_c / 1.5\, P_c)$ i.e., 66.66%.

The concept of power saving will become clear after solving the following example.

EXAMPLE 3.13. Calculate the percent power saving for a DSB-SC signal for the percent modulation of (*a*) 100% and (*b*) 50%

Solution :

1. Power saving in the DSB-SC signal

The total power in AM wave, $P_1 = P_c \left[1 + \frac{m^2}{2}\right]$

At 100% depth of modulation $m = 1$

$$\therefore \quad P_1 = 1.5\, P_c$$

$$\therefore \quad \% \text{ power saving} = \frac{P_c}{1.5\, P_c} = 66.66\%$$

At 50% depth of modulation $m = 0.5$

$$\therefore \quad P_t = 1.125\, P_c$$

$$\therefore \quad \% \text{ power saving} = \frac{P_c}{1.125 P_c} = 88.88\%. \qquad \textbf{Ans.}$$

2. Transmission Bandwidth

The transmission bandwidth of DSB-FC with a single tone modulation is $2\, f_m$ where f_m is the frequency of the modulation signal. Thus, transmission bandwidth of DSB-SC is same as that of the standard AM wave.

3. How will you differentiate between DSB-FC and DSB-SC signals ?

The time domain displays of DSB-FC signal with $m = 100\%$ and DSB-SC signal look exactly the same. The only difference between them is that in the time-domain display of DSB-SC, the carrier undergoes 180° phase shift. This is how we can identify the DSB-SC signal.

3.27.4. Coherent Detection For Single Tone DSB-SC Wave

1. Mathematical Expression

The coherent detection discussed earlier can be used for the detection of a single tone DSB-SC wave.

The output of product modulator is given by,

$$m(t) = s(t) \times c(t) \qquad \text{... (3.104)}$$

where $s(t)$ = DSB-SC wave = $\frac{1}{2} V_m V_c\, [\cos 2\pi\, (f_c + f_m)\, t + \cos 2\pi\, (f_c - f_m)\, t]$

and $c'(t)$ = Local carrier = $\cos\, (2\pi\, f_c\, t)$

Substituting the expressions for $s(t)$ and $c'(t)$, we get,

$$m(t) = \cos\, (2\pi\, f_c\, t) \left\{\frac{1}{2} V_m V_c \cos[2\pi (f_c + f_m) t] + \frac{1}{2} V_m V_c \cos[2\pi (f_c - f_m) t]\right\}$$

$$\therefore \quad m(t) = \frac{1}{2}\, V_m\, V_c \cos\, [2\pi\, (f_c + f_m)\, t]\, .\, \cos\, (2\pi\, f_c\, t) + \frac{1}{2}\, V_m\, V_c \cos\, [2\pi\, (f_c - f_m)\, t]\, \cos\, (2\pi\, f_c\, t) \qquad \text{... (3.105)}$$

* Both DSB and SSB signals are more efficient in terms of power usage. The power wasted in the useless carrier is saved, thereby allowing more power to be put into the sidebands.

But, $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$$\therefore \quad m(t) = \frac{1}{4} V_m V_c \{\cos[2\pi(2f_c + f_m)t] + \cos(2\pi f_m t)\} + \frac{1}{4} V_m V_c$$

$$\{\cos[2\pi(2f_c - f_m)t] + \cos(2\pi f_m t)\} \quad \ldots (3.106)$$

The terms having frequencies $(2f_c + f_m)$ and $(2f_c - f_m)$ are removed by the low pass filter which follows the product modulator. But, the terms having frequency f_m are passed through to the output.

2. Output Voltage

$$\text{Output voltage } v_0(t) = \frac{1}{2} V_m V_c \cos(2\pi f_m) t) \quad \ldots (3.107)$$

Thus, the scaled message signal is obtained at the detector output.

3.28 SINGLE SIDEBAND SUPPRESSED-CARRIER (SSB-SC) MODULATION *(Important)*

1. Definition

As discussed earlier that amplitude modultion and double-sideband suppressed-carrier (DSB-SC) modulation are wasteful of bandwidth since they both need a transmission bandwidth equal to twice the message signal bandwidth. In either case one half of the transmission bandwidth is occupied by the upper sideband of the modulated signal whereas the other half is occupied by the lower sideband. However, the lower and upper sidebands are uniquely related to each other by virtue of their symmetry about the carrier frequency, *i.e.*, if amplitude and phase spectra of either sideband is given, we can uniquely determine the other. This means that as far as the transmission of information is concerned, only one sideband is necessary. Thus, if the carrier and one of the two sidebands are suppressed at the transmitter, no information is lost. Modulation of this type which provides a single sideband with suppressed carrier is known as single sideband suppressed carrier (SSB-SC) system. Thus, SSB-SC system reduces the transmission bandwidth by half. This means that in a given frequency band we can accommodate twice the number of channels by using a single sideband in place of both the sidebands*.

2. Frequency Spectrum

Figure 3.38 further illustrate the concept of single sideband modulation with the help of different frequency spectrums.

Figure 3.38(*a*) shows the frequency spectrum of modulating or baseband signal. It contains ω_m as the maximum frequency component. Figure 3.38(*b*) shows the frequency spectrum of a DSB-SC modulation which contains no carrier but two sidebands, *i.e.*, lower sideband and upper sideband. Figure 3.38(*c*) shows the frequency spectrum of single sideband suppressed carrier modulation consisting of upper sideband only, *i.e.*, lower sideband and carrier signal are suppressed. Figure 3.38(*d*) shows the frequency spectrum of single sideband suppressed carrier modulation consisting of lower sideband only, *i.e.*, upper sideband and carrier signal are suppressed.

REMEMBER

For SSB transmissions, it does not matter whether the upper or lower sideband is used, since the information is contained in both.

* An AM signal with no carrier and one sideband is called a single sideband (SSB) signal. The upper and lower sidebands contain the same information, and one is not preferred over the other.